World - sheet Instantons in the Theory of the QCD Strings

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Abstract

The QCD string is manifested through the world-sheet instanton solutions which are responsible for confinement phenomenon and construction of θ-vacua.

Keywords: QCD string, Instanton, Tension, β-function, Semiclassics, Tachyon.

1 Introduction

This article is a continuation of my attempts to realize one possibility for extracting of the effective string dynamics from the strong-coupling regime of $SU(N)$ gauge theory at a large number N of colors. In past decade I had got the effective string picture with quantization of string action [1]. At this time it was looked as a some strange result. Now I understand that it is a manifestation of the world-sheet instantons which are responsible for confinement phenomenon and construction of θ-vacuum.

Let us regard the nonperturbative calculation of hadron-field correlation functions

$$K(1,\ldots,n) = \langle \Psi^+(x_1)\Psi(x_1)\cdots\Psi^+(x_n)\Psi(x_n)\rangle \tag{1}$$

in the framework of the $SU(N)$ gauge field theory in the limiting case

$$N \gg Ne^2 \gg 1 \tag{2}$$

(e is the gauge coupling constant).

Here,

$$\Psi^+(x)\Psi(x) \equiv \sum_{c=1}^{N} \Psi_c^+(x)\Psi_c(x)$$

is a color-singlet operator of a composite point-like meson, and $\Psi_c(x)$ is a scalar (for the sake of simplicity) quark field, which is a spinor in the color space.

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The hadron-field correlation function is

$$K(1,\ldots,n) = B^{-1}\int d\mu[A]D\Psi D\Psi^+ e^{iS_{YM}[A,\Psi,\Psi^+]} \times \{\Psi^+(x_1)\Psi(x_1)\cdots\Psi^+(x_n)\Psi(x_n)\},$$

where B is a normalization constant. Integrating with respect to Ψ and discarding internal quark loops, which involve small factors of order $O(1/N)$, we obtain the expression for the connected part of $K(1,\ldots,n)$. It contains the Wilson loop and has the form

$$K(1,\ldots,n) \approx \int Dx_\mu(\gamma)D\lambda(\gamma)\langle C(\Gamma)\rangle_A e^{i\oint\limits_\Gamma d\gamma\frac{1}{2}(\dot{x}^2/\lambda+\lambda m_0^2)}.$$

Here, $\gamma(0\le\gamma\le 1)$ is a variable specifying the parametrization of the contour Γ that passes through the fixed points $x_1,\ldots,x_n$ (see the figure);

$$\dot{x}_\mu = \frac{dx_\mu}{d\gamma}, \quad x_\mu(1) = x_\mu(0);$$

m_0 is the bare quark mass; $\lambda(\gamma)$ is the one-dimensional metric on Γ; and

$$\langle C(\Gamma)\rangle_A = B^{-1}\int d\mu[A]e^{iS_{YM}[A]}C(\Gamma), \tag{3}$$

$$C(\Gamma) = \mathrm{tr}\left[P\exp\left(ie\oint\limits_\Gamma dx^\mu A_\mu\right)\right]. \tag{4}$$

We used the first quantization representation for propagator of scalar quark

$$\Delta\left[x(\gamma_2);x(\gamma_1)\right]_{cc'} = \int\limits_{x(\gamma_1)}^{x(\gamma_2)} Dx_\mu(\gamma)\int D\lambda(\gamma) \times e^{i\int\limits_{\gamma_1}^{\gamma_2} d\gamma\frac{1}{2}(\dot{x}^2/\lambda+\lambda m_0^2)}\left\{P\exp\left[ie\int\limits_{\gamma_1}^{\gamma_2} d\gamma\frac{dx_\mu}{d\gamma}A_\mu\right]\right\}_{cc'}.$$

Let us introduce auxillary Grassman variables $\xi_c(\gamma)$ $(c = 1,\ldots,N)$ describing color spin of quarks ($\xi_c(\gamma)$ is a color spinor and a scalar with respect to the Lorentz group). Using these variables, we can recast $C(\Gamma)$ into the form of the path integral

$$C(\Gamma) = \int D\xi(\gamma)D\bar{\xi}(\gamma)\exp\{-iS[\xi,\bar{\xi}] - \bar{\xi}_d(0)\xi_d(0)\}\xi_c(1)\bar{\xi}_c(0), \tag{5}$$

where

$$S[\xi,\bar{\xi}] = i\oint\limits_\Gamma d\gamma\bar{\xi}_c(\gamma)\left[\frac{d}{d\gamma} - ieA_\mu\dot{x}^\mu\right]_{cd}\xi_d(\gamma). \tag{6}$$

As a result of this transformation, the correlation function $K(1,\ldots,n)$ assumes the form

$$\begin{aligned} K(1,\ldots,n) =& B^{-1}\int d\mu[A]Dx(\gamma)D\lambda(\gamma)D\xi(\gamma)D\bar{\xi}(\gamma) \\ &\times \xi_c(1)\bar{\xi}_c(0)\exp\{iS[A,\xi,\bar{\xi},x] - \bar{\xi}_d(0)\xi_d(0)\}, \end{aligned} \tag{7}$$

where total action is

$$S[A,\xi,\bar{\xi},x] = -\frac{1}{4}\int d^4x G^a_{\mu\nu}G^{a,\mu,\nu} + \oint\limits_{\Gamma} d\gamma\left[\frac{1}{2}(\dot{x}^2/\lambda + \lambda m_0^2) + i\bar{\xi}D_\gamma\xi\right], \tag{8}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + ef^{abc}A^b_\mu A^c_\nu, \tag{9}$$

and D_γ is the covariant derivative with respect to the parameter γ.

2 The semi-classical $1/N$ approximation

To calculate the correlation function (1), we will use the semi-classical $1/N$ expansion. It was shown in [2] that, in the limit of large N, the parameter $1/N$ plays the same role as Planck's constant $\hbar$ in the ordinary semi-classical approximation of quantum mechanics. In the leading order in N, we need take into account only the contributions of planar gluon diagrams. This means that, in the path integral $\langle C(\Gamma)\rangle_A$ (3), it is necessary to carry out summation over the subset of gauge fields whose contributions are greatest in this approximation. We use topological arguments to single out this subset of fields. In order that the semi-classical $1/N$ approximation be valid, it is necessary to have a stable self-consistent extremal of the fields $\{\xi, A\}$. Studies in the field of instanton and soliton physics showed that stable field configurations in nonlinear field theories correspond to topologically nontrivial solutions of classical equations of motion.

The variation of the action functional (8), that is, the condition $\delta S/\delta\bar{\xi}_c = 0$, gives the equation of motion

$$\frac{d\xi_c}{d\gamma} - ie(A_\mu)_{cd}\xi_d\dot{x}^\mu = 0. \tag{10}$$

The formal solution of this equation is

$$\hat{\xi}_c(\gamma) = \left[P\exp\left(ie\int_0^\gamma d\gamma'\dot{x}^\mu A_\mu\right)\right]_{cd}\xi_d(0), \tag{11}$$

where $\xi_d(0)$ is arbitrary. The ordered exponential function in (11) is defined on the contour Γ and maps it into the $SU(N)$ group. This mapping is trivial for $N \geq 2$ because the homotopic group $\pi_1[SU(N)] = 0$. The only exception arises when a quasi-Abelian field appears in expression (11); in this case, we are dealing with mapping of Γ into the subgroup $U(1)$ of the $SU(N)$ group. This mapping is nontrivial because we have $\pi_1[U(1)] = Z$, where Z is the group of integers. Further, the field (11) will be a true classical solution of equation (10) if it is single-valued on the closed contour Γ. This leads to the quantization of the chromoelectric flux of the quasi-Abelian field through an arbitrary surface Σ with the boundary $\partial\Sigma = \Gamma$ [3]. This flux quantization can stabilize the field configuration $\{\hat{\xi}, \hat{A}\}$ with respect to fluctuations if we are dealing with the full flux , and not with some part of the taken at random. Because the contour Γ forms a one-dimensional boundary, this condition can be satisfied for a quasi-two-dimensional field of the form

$$A^a_\mu(x(z))\bigg|_\Sigma \equiv \frac{\partial x_\mu}{\partial z^i}A^{a,i}(z)\bigg|_\Sigma, \tag{12}$$

which is defined on the surface Σ with the boundary $\partial\Sigma = \Gamma$. Here,

$$i = 1,2; \qquad \mu = 1,2,3,4; \qquad a = 1,\ldots,N^2-1; \tag{13}$$

and the equation $x_\mu = x_\mu(z)$ determines the embedding of the surface Σ into the flat space $R^{1,3}$ (or R^4 in the Euclidean formulation). Thus, the topologically nontrivial configuration $\{\xi[A], A\}$ is realized only in the subset of quasi-two-dimensional fields A and the semi-classical $1/N$ calculations should be restricted to the evaluation of contributions from this subset of fields (12). In principle, there are contributions from other (four-dimensional) fields, but they do not give stable extremals in our problem. Therefore, these contributions cannot be calculated with the aid of the semi-classical expansion, and some other methods must be designed to accomplish these ends. However, there is independent evidence that, in the leading order of the strong - coupling approximation, the string tension is dominated by contributions from two-dimensional fields.This statement follows from a comparison of the string tension that is calculated in the Hamiltonian formulation of the lattice gauge theory [4] with the result obtained by computing Wilson loop in the Yang-Mills theory on arbitrary (two- dimensional) surfaces [5]. These results also coincide with the string tension (26) obtained in this study. The coincidence of these results shows that the semi-classical $1/N$ approximation includes the leading contributions to hadron correlators in the strong - coupling regime. So, if we take into account only contributions of fields type (12), the calculation of the expectation value in (7) is reduced to integration with respect to the two-dimensional field $[A_i^a(z)]_\Sigma$ for a fixed surface Σ and subsequent summation over surfaces Σ.

Going over to the new variables, we obtain the effective two-dimensional action for each surface Σ [6].

$$\begin{aligned} S_\Sigma^{eff}[A, \xi, x, \lambda] = & -\frac{1}{4}\int\limits_\Sigma d^2z(-h)^{\frac{1}{2}} h^{il} h^{kn} G_{ik}^a G_{ln}^a \\ & + \oint\limits_{\partial\Sigma} d\gamma \frac{1}{2}(\dot{x}^2/\lambda + \lambda m_0^2) + i \oint\limits_{\partial\Sigma} d\gamma \bar{\xi}_c \left(\frac{d}{d\gamma} - i\varepsilon A_i \dot{z}^i\right)_{cd} \xi_d, \end{aligned} \tag{14}$$

$$G_{ik}^a = \partial_i A_k^a - \partial_k A_i^a + \varepsilon f^{abc} A_i^b A_k^c. \tag{15}$$

Here, $\varepsilon = e/\delta$ is the two-dimensional charge, δ is the UV regulator with the dimension of length, $h = det(h_{ik})$, and

$$h_{ik} = \frac{\partial x_\mu}{\partial z^i}\frac{\partial x^\mu}{\partial z^k}$$

is the metric induced on the surface Σ by embedding $x_\mu = x_\mu(z)$.

3 World-sheet instantons as a solutions of the classical equations

The condition $\delta S^{eff}/\delta A_i^a = 0$ gives the equation of motion

$$\partial_i(\sqrt{-h} G^{a,ik}) - \varepsilon f^{abc} \sqrt{-h} G^{b,ik} A_i^c = 0 \tag{16}$$

for the field $A^{cl}(z) \equiv \hat{A}(z)$ on the surface Σ and the boundary condition

$$\sqrt{-h} G^{a,ik}(z(\gamma)) e_{si} \dot{z}^s = \varepsilon T^a(\gamma) \dot{z}^k \tag{17}$$

on $\delta\Sigma = \Gamma$. The boundary condition takes into account the presence of quarks at the boundary of the surface. Here, e_{si} is the antisymmetric unit tensor, and

$$T^a(\gamma) = \bar{\xi}_c(\gamma) \left(\frac{\lambda^a}{2}\right)_{cd} \xi_d(\gamma) \tag{18}$$

is the operator of the quark color spin. By virtue equation (10), the operator $T^a(\gamma)$ is a covariantly constant quantity. Equation (16), together with the boundary condition (17), have a solution of the form

$$G^{a,ik}(z) = \varepsilon \left(e^{ik}/\sqrt{-h(z)}\right) I^a(z), \tag{19}$$

provided that $I^a(z)$ satisfies the equation

$$D_i^{ab} I^b(z) = 0. \tag{20}$$

Tensor (19) also obeys the two-dimensional Bianchi identity. In view of boundary condition (17), the square of the vector I^a equals the square of the quark color spin; that is

$$I^2 = I^a I^a = (N^2 - 1)/2N. \tag{21}$$

In a special gauge where

$$I^a = \text{const}, \tag{22}$$

the potential $\hat{A}$ corresponding to (19) has the form

$$\begin{aligned} \hat{A}_i^a(z) =& I^a a_i(z)/\varepsilon, \\ a_i(z) =& \frac{e_{ik}}{2} \sqrt{-\hat{h}} \hat{h}^{kl} \partial_l \ln \sqrt{-\hat{h}} \end{aligned} \tag{23}$$

where $\hat{h}$ is the metric of the constant curvature

$$R = -2\varepsilon^2 \tag{24}$$

on Σ. Substituting the self-consistent configuration $\{\hat{A}, \hat{\xi}\}$ into the action (14) and going over to the Euclidean space, we reduce S^{eff} to the following form of the string action with constraints (only in Euclidean space above action has a right sign):

$$\begin{aligned} S_\Sigma^{eff}[x, \lambda] =& k_0 \int_\Sigma d^2 z \sqrt{h(x(z))} \\ &+ \frac{1}{2} \oint_{\Gamma = \partial\Sigma} d\gamma \left[\dot{x}^2(z(\gamma))/\lambda(\gamma) + \lambda(\gamma) m_0^2\right] + \text{constraints}, \end{aligned} \tag{25}$$

where

$$\dot{x}_\mu = \frac{dx_\mu}{d\gamma} = \frac{dx_\mu}{dz^i}\frac{dz^i}{d\gamma}.$$

The bare string tension is given by

$$k_0 = \frac{e^2}{2\delta^2}\left(\frac{N^2 - 1}{2N}\right), \quad e^2 = e_0^2/N, \quad e/\delta \equiv \varepsilon. \tag{26}$$

(The string tension k is renormalized as the result of subsequent summation over surfaces. In this procedure, the parameter δ is made to tend to zero; this is equivalent to the removal of the cut-off and the introduction of the normalization point.) In the limit $N \to \infty$, k_0 is independent of N; that is, we have $S^{eff}[\hat{A}, \hat{\xi}] \sim O(1)$. The constraints that enter in formula (25) ensure that our solutions will be the world-sheet instantons: a topologically nontrivial embedding of the world sheet in target space.

4 Constraints

The first constraint imposes condition of quantization on the string action; that is,

$$k_0 \int\limits_{\Sigma} d^2 z \sqrt{h} = \pi |Q|, \quad Q = 0, \pm 1, \pm 2, \dots \tag{27}$$

This condition is a corollary of the quantization of the chromoelectric flux on the world sheet Σ. Flux quantization follows from the requirement that the classical solution $\hat{\xi}_c(\gamma)$ (11) defined on the closed contour $\Gamma = \partial\Sigma$ be single-valued. The index Q in (27) determines the total increment of the phase of $\hat{\xi}(\gamma)$ upon the circumvention along the contour Γ. This index has the gauge invariant representation

$$Q = \frac{\varepsilon}{4\pi} \int\limits_{\Sigma} d^2 z e^{ik} I^a(z) G^a_{ik}(z) \tag{28}$$

and characterizes various topological sectors. In the special gauge (22), (23), it assumes the simple form

$$Q = \frac{I^2}{2\pi} \oint\limits_{\Gamma} dz^i a_i. \tag{29}$$

Furthermore, it can be shown [3] that the gauge component of the action(14) is bounded from below, namely,

$$S_{YM}[A] \geq \pi |Q|. \tag{30}$$

As a result, the string field configuration $\{\hat{A}, \hat{\xi}\}$ turns out to be stable to small fluctuations of the gauge field that refer to a definite sector specified by Q.

The second constraint is associated with condition (24), which requires that the scalar curvature be constant. If condition (24) were not imposed, it would be possible to contract the boundary $\partial\Sigma$ of the surface Σ to zero by deforming the surface without changing the surface area $A(\Sigma)$. This would destroy the topological classification (27) with $Q \neq 0$. Moreover, condition (24) ensures the $1/N$ - suppression of Gaussian fluctuations δA [7]. The solution of (23) expressed in terms of metric $\hat{h}$ actually associates the spin connection on the surface Σ with the vector potential $a_i(z)$ or, which is equivalent, associates the curvature tensor with the strength tensor $F_{ik} = \partial_{[i} a_{k]}$ on Σ. Thus, in the approach developed here, the geometry of the string world sheet is determined by the dynamics of the gauge field on the same sheet. So, we have the manifestation of world-sheet instantons - topologically nontrivial embedding of the string sheet in outer space.

5 The partition function and correlation functions

In view of the quantization condition (27), the partition function

$$Z = \int Dx_\mu(z) D\lambda(\gamma) \exp\{-S[x(z), \lambda(\gamma)\} \tag{31}$$

is decomposed into the sum of contributions from all topological sectors; that is, we have

$$Z = \sum_{|Q|=0}^{\infty} Z_{|Q|} = \sum_{|Q|} \left(Z_{Q^+} + Z_{Q^-} \right),$$
$$Q^{\pm} = \pm\, |Q|. \tag{32}$$

The calculation of Z_Q is carried out with the aid of the relation

$$\int Dx_\mu \exp\left[-k_0 \int\limits_\Sigma d^2z\sqrt{h}\right] \mathrel{\hat{=}} \int Dx_\mu Dg_{ab}$$
$$\times \exp\left(-\frac{k_0}{2}\int\limits_\Sigma d^2z\sqrt{g}g^{ab}\partial_a x_\mu \partial_b x_\mu\right), \qquad (33)$$

which holds only in the leading approximation of the of steepest descent. (The symbol $\hat{=}$ means that the integral with respect to g_{ab} equals the value of the integrand taken at the saddle point.)

The calculation of the partition function Z by means of integration the conformal anomaly [8] with allowance for the constraints showed [9] that the partition function Z_Q is expressed only in terms of the Euler characteristic χ of the world sheet, that is,

$$Z_{|Q|} \sim \exp\left[-D(\chi/6 + 1/3)|Q|\right]. \qquad (34)$$

This means that, in the approximation used here, the $SU(N)$ gauge theory is reduced to a topological theory. The conformal anomaly proportional to the Liouville action functional is neutralized by the condition R = constant: that is, there is no critical dimension $D = 26$.

In the calculation of Z (31), there arise divergences proportional to the surface area and perimeter of the world sheet. This enable us to renormalize the string tension k and the quark mass m_q [9]. The renormalized mass of a quark determines the geodesic curvature of the boundary $\partial\Sigma$; that is, $\kappa_{\hat{g}} = m/4$. In particular there is relation

$$\kappa_{\hat{g}}^2 = -R/2 \qquad (35)$$

which is a characteristic of the Beltrami pseudo-sphere, that is, our instanton wraps around noncontractible surface in target space that is pseudo-sphere. We also have the relation $k \sim m^2 \sim R$.

Thus, there is only one independent dimensional parameter, namely, the renormalized string tension

$$k \equiv \frac{1}{2\pi\alpha'} = \frac{e^2(a)}{2a^2}\left(\frac{N^2-1}{2N}\right). \qquad (36)$$

Its value can be determined by analyzing experimental data that concern distances a on the order of the confinement radius ($a \sim r_{conf}$). Here, $e(a)$ is the running coupling constant, and a is the normalization point. In the nonperturbative analysis of a field theory that admits quark confinement, it is natural to use the renormalization scheme in which the string tension k is fixed:

$$\frac{dk}{da} = 0. \qquad (37)$$

(The quantity $\sqrt{k}$ plays a role here similar to that of the dimensional perturbative parameter Λ_{QCD}; these two quantities are related by a linear equation.) From condition (37) and from equation (36), it follows that the Gell-Mann-Low function has the form

$$\beta(e) \equiv -a\frac{de}{da} = -e(a). \qquad (38)$$

This expression coincides with the first term of the expansion of the β-function in inverse powers of the charge e in the strong-coupling approximation of the Hamiltonian formulation of the lattice gauge theory [4]. This formulation is convenient because it permits the separation of contributions from electric and magnetic color fields. In the strong-coupling approximation, the contribution of the electric field is dominant. At distances $a \ll r_{conf}$, where the charge is small, it is necessary to take into account the contribution of the magnetic field as well. Numerical calculations showed [4] that, in this case, expression (38) goes over smoothly to the standard formulas

$$\beta(e) = -b_0 e^3 - b_1 e^5 \ldots,$$
$$b_0 = \frac{11}{48\pi^2} N, \quad b_1 = \frac{34}{3}\left(\frac{N}{16\pi^2}\right)^2 \tag{39}$$

for $\beta(e)$ in the weak-coupling approximation.

The final stage of the calculation of hadron-field correlation functions is as follows [10]. In the approximation of scalar quarks, the above functions are reduced to Koba-Nielsen dual resonance amplitudes in each topological sector. It turned out that the above-listed distinctions of the chromoelectric string from the standard model manifest themselves mainly in the calculation of the partition function and drop out,to a considerable extent, of final expressions for correlation functions. However, there is still a tachyon in the ground state in each topological Q-sector. An infinite number of degenerate Q-sector enables us to consider a new possibility for solving the long-standing tachyon problem in dual resonance models. Namely, the introduction of the θ-vacuum,which is a superposition of these sectors, is expected to result in a shift in the mass spectrum and lead to the elimination of the tachyon state from the theory.

6 Homotopical classification of backgrounds, θ-vacuum and tachyon mass problem

The solution of the equation of motion (10) according to (11), (23) and [6] has a form

$$\xi_c(\gamma) = e^{-i\phi(\gamma)} \xi_c(0) \tag{40}$$

where the phase

$$\phi(\gamma) = I^2 \int_0^\gamma d\gamma' \frac{dz^i}{d\gamma'} a_i(z) = I^2 \int_0^\gamma d\gamma' a_{\gamma'} \tag{41}$$

does not depend on the index $c = 1, \ldots, N$ this being a consequence of the spontaneous breaking of the $SU(N)$ symmetry to $U(1)$. The phase $\phi(\gamma)$ forms mappings $S_1 \to S_1$, characterized by a winding number Q (28), (29). Then we can construct the backgrounds (pure gauge) at the boundary Γ for each sectors Q

$$a_\gamma = i\Lambda_Q^{-1} \frac{\partial}{\partial\gamma} \Lambda_Q = \frac{2\pi Q}{I^2}, \tag{42}$$

where

$$\Lambda_Q(\gamma) = e^{-i2\pi Q\gamma/I^2}, \tag{43}$$

and

$$\phi(\gamma=1) \equiv I^2 \oint\limits_{\Gamma} d\gamma a_\gamma = 2\pi Q. \tag{44}$$

Because the our world-sheet instanton has topological index Q=1, it can makes the communication between next backgrounds. Anticipating that some tunnelling does take place, the correct vacua will be linear combinations of the topological vacua and are given by

$$|\theta\rangle = \sum_{Q=-\infty}^{\infty} e^{i\theta Q}|Q\rangle. \tag{45}$$

It is equivalent to add the θ -term to the action [11]

$$S_\theta = S_0 + i\theta Q. \tag{46}$$

Each tachyon for sector Q has the statistical weight is equal to (for $D=4$) [9]

$$(\beta A_{eff})^{|Q|}\frac{e^{-\frac{4}{3}|Q|}.e^{i\theta Q}}{|Q|!}, \quad S_0 = \frac{4}{3}|Q| \tag{47}$$

$$(\text{The Euler characteristic } \ \chi = 0 \ \text{ for pseudo} - \text{sphere}).$$

The pre-exponential factor taking into account the contribution from Gaussian fluctuations of gauge field around the instanton solution. According to [7] this factor is proportional to effective instanton area $A_{eff} = 2\pi a^2$, where a-radius Beltrami pseudo-sphere. The $|Q|!$ arise because of indistinguishability of instantons.

Let us regard the contribution of one tachyon pole in the dual resonance amplitude $A(s,t)$. It is equal to

$$\frac{1/\alpha'}{m_0^2 - s}, \quad m_0^2 < 0. \tag{48}$$

Then one makes the transformation

$$\int\limits_{-\infty}^{\infty} \frac{e^{-isA_M}}{m_0^2 - s - i\varepsilon} = e^{-im_0^2 A_M} \to e^{-m_0^2 A_E} \tag{49}$$

where we have came from Minkovski area space A_M to Euclidean area $A_E = A_{eff}$. If we multiply this contribution by factor (47) and make the sum over all topological sectors with n instantons and $\bar{n}$ anti-instantons, we will get to result

$$\begin{aligned} &\frac{1}{\alpha'}e^{-m_0^2 A_{eff}} \sum_{n,\bar{n}=0}^{\infty} \frac{\left(\beta A_{eff} e^{-S_0 + i\theta}\right)^n}{n!} \frac{\left(\beta A_{eff} e^{-S_0 - i\theta}\right)^{\bar{n}}}{\bar{n}!} \\ &= \frac{e^{-m_0^2 A_{eff}}}{\alpha'} \exp\left\{e^{-S_0}\beta A_{eff} 2\cos\theta\right\} = \frac{1}{\alpha'}e^{-m^2(\theta)A_{eff}}, \end{aligned} \tag{50}$$

where

$$m^2(\theta) = m_0^2 - 2\beta e^{-S_0}\cos\theta. \tag{51}$$

So, we have got the shift of the tachyon mass.

Let us stress also that θ-term in (46) does not violate the parity symmetry because contains only the internal two-dimensional potential. Also there is not any violation of the charge conjugation symmetry, because θ-term contains charge squared according to (28), (19).

7 Conclusion

In the leading order of the semi-classical $1/N$ expansion, the partition function and correlation functions are apparently dominated by the world-sheet instantons solutions. This ensures the string picture with quark confinement and introducing of the θ-vacua.

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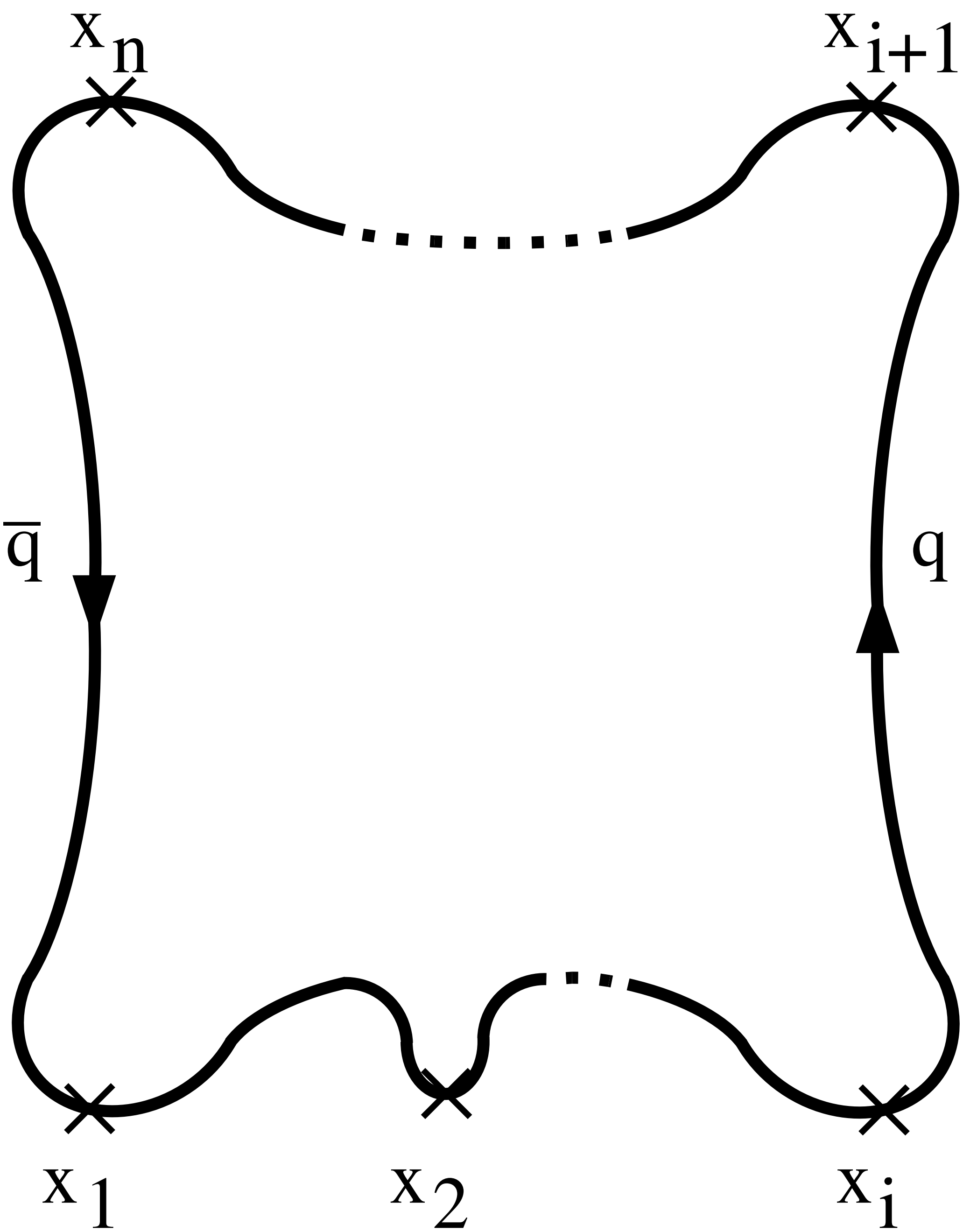

Figure 1: The contour Γ in 4-dim, external space corresponding to the connected part of the correlator $K(x_1, \ldots, x_n)$.